Effects of radiative corrections on a quantum disordered self-dual Josephson junction array

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Abstract I study the quantum effective potential of a U(1)-invariant theory involving complex scalar fields coupled to topologically massive gauge fields and gapless fermions relevant to the low energy dynamics of self-dual Josephson junction arrays. It is found that gauge-boson contribution to the one-loop effective potential does not induce symmetry breaking, with the result that the insulating phase in this model is stable against gauge-boson fluctuations. However, coupling to gapless fermions induces a correction term in the effective potential which changes the symmetry of the ground state and favors transitions between a superconducting state and an insulating state.

Keywords Josephson junction array, quantum effective potential, Chern-Simons gauge theory.

Abbreviations JJA, Josephson junction array; SSB, Spontaneous symmetry breaking.

Gauge theories in (2+1)-dimensional space-time have proven to be of significant interest in planar condensed matter systems. The gauge fields in these systems materialize from some general considerations such as current conservation constraints $\partial_{\mu} J_{\mu} = 0$, which in (2+1) dimensions can be represented as the curl of a vector potential $J_{\mu} = \epsilon_{\mu\nu\rho} \partial_{\nu} a_{\rho}$. The resulting low energy action is gauge invariant, reflecting the original gauge invariance of the matter currents and may contain terms such as the Chern-Simons terms describing incompressible quantum fluids and/or Maxwell or Meissner terms describing superconductors.

Here I analyze the quantum effective potential of a three-dimensional abelian Maxwell-mixed-Chern-Simons gauge theory which is relevant to the physics of self-dual Josephson junction arrays (JJA) [1]. In these planar systems, the currents of Cooper pairs $J_{\mu}$ and of vortices $\ell_{\mu}$ are conserved and can be expressed in terms of fictitious gauge fields $\ell_{\mu} = \epsilon_{\mu\nu\rho} \partial_{\nu} a_{\rho}$, $J_{\mu} = \epsilon_{\mu\nu\rho} \partial_{\nu} b_{\rho}$. In this formulation, a mixed Chern-Simons term $a_{\mu} \epsilon_{\mu\nu\rho} \partial_{\nu} b_{\rho}$ describes both the Lorentz force exerted by the vortices on the charges and the magnus force exerted by the charges on the vortices [2]. The low energy dynamics of self-dual JJA is captured by complex
fields associated with disordering due to electric charges and magnetic charges minimally coupled to \(a_\mu\) and \(b_\mu\) [3]. The condensation of these fields establish the quantum phases of the related JJA system, specifically ground states with nonzero condensates of the charge disorder fields describe superconducting phases whereas zero condensates describe insulating phases.

Here I investigate the spontaneous symmetry breaking of the U(1) gauge symmetry by radiative corrections. This is essential since the classical potential may be unstable under these corrections. A well-known example is the seminal study of Coleman and Weinberg [4] who showed that in four-dimensional massless scalar QED radiative corrections to the effective potential lead to SSB. The low energy effective theory of self-dual JJA is described by the following Euclidean Lagrangian [3]

\[
L = \frac{1}{4g_1} f_{\mu\nu}^2 + \frac{1}{4g_2} g_{\mu\nu}^2 + i\eta \partial_\mu \epsilon^{\mu\nu\rho\sigma} \partial_\rho b_\sigma + \left( i\partial_\mu - a_\mu \right) \phi^2 + \left( i\partial_\mu - b_\mu \right) \chi^2
\]

\[+ V(\phi, \chi) + \bar{\psi} (i\gamma_\mu (i\partial_\mu - a_\mu)) \psi + G \phi^* \phi \bar{\psi} \psi \]

(1)

where \(f_{\mu\nu} = \partial_\mu b_\nu - \partial_\nu b_\mu\), \(g_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu\) and \(g_1, g_2\) are related to the Josephson and to the charging energies characterizing JJA systems \((g_1=4\pi^2 E_J, g_2=8E_C)\) [3].

The complex fields \(\phi\) and \(\chi\) describe quantum disordering due to electric and magnetic excitations in the system. The tree level potential can be expanded in power of the fields \(V(\phi)=r\phi^2+\lambda\phi^4...,\) and describes the short distance physics contained in the microscopic model on the lattice. The fermionic part in Eq. (1) describes quantum disordering to fermions and is relevant when the JJA system is connected to a reservoir of gapless single-particle excitations [1]. Here the coupling between gapless quasiparticles and JJA is achieved by invoking the general principles of gauge invariance and renormalization. Another common coupling in JJA is due to dissipation which tends to suppress quantum phase fluctuations [1] and enhances the charge fluctuations thereby stabilizing the superconducting phase. This type of dissipative has been considered recently [5] in a tunneling contact with a graphene layer which acts as a source of gapless quasiparticles. It was reported that a coupled JJA to a graphene substrate with gapless excitations further enhances charge fluctuations favoring superconductivity.
The main goal is to derive from Eq. (1) the quantum effective potential and to analyze its symmetry-breaking minima which describe phases where topological electric or magnetic excitations proliferate and condense. We briefly sketch the separate contributions to the one-loop effective potential first arising from the gauge fields followed by that due to the fermions. More details about the formalism, derivation and analysis will be published elsewhere.

After shifting the fields \( \chi \rightarrow \chi + v_1 \), \( \phi \rightarrow \phi + v_2 \) and including the gauge fixing terms, we integrate out the quadratic part in the gauge fluctuations. The required gauge-boson propagators are

\[
D^{\mu \nu}_{ab}(q) = \frac{1}{q^2 + \Delta^2},
\]

and

\[
D^{ab}_{\mu \nu}(q) = \frac{\eta}{\Delta} e_{\mu \nu} q_{\lambda}, 
\]

\[
\delta^T_{\mu \nu} = \frac{q_{\mu} q_{\nu}}{q^2}, \quad \Delta_i = q^2 / g_i + v_i^2 \text{ and } \Delta = \Delta_1 \Delta_2 + \eta^2 q^2.
\]

After eliminating a linearly divergent quadratic term by an appropriate counter term, the finite correction to the effective potential is found to be

\[
V_G = \frac{1}{12 \pi} \left( \eta^2 g_1 g_2 + g_1 v_1^2 + g_2 v_2^2 - v_1 v_2 \sqrt{g_1 g_2} \right) \left( \frac{v_1}{\sqrt{g_1}} + v_2 \sqrt{g_2} \right)^2 + \eta^2 g_1 g_2 \quad (2)
\]

Inspection of this contribution shows clearly that it doesn't favor SSB. Hence in the absence of a SSB term at the tree level, radiative corrections due to the fluctuations in the gauge fields do not change the symmetry of the ground state. We conclude that insulating phases described by zero condensates in this model are stable against gauge fluctuations.

Next, I include the effect of quantum disordering due to gapless fermions which turn out to change completely the symmetry of the ground state. The one-loop contribution of the fermions has a linearly divergent \((\phi^* \phi)^2\) term which can be eliminated by an appropriate counter term and a finite contribution given by

\[
V_F = (G^3 / 6 \pi ) (\phi^* \phi)^3.
\]

It is worth noting that the fermionic contribution comes with a positive sign as opposed to the one from gauge fluctuations which comes with a negative sign. Combining the two contributions leads to the possibility of a nontrivial vacuum expectation value even in the absence of tree level SSB terms. This can be easily seen by taking \( < \chi > = 0 \) and solving the gap equation \( dV_{\text{ren}} / d\phi = 0 \) with the re-scaled potential \( V_{\text{ren}} = 1 - (\phi^2 + 1)^{3/2} + (r + 3/2)\phi^2 + (\lambda + 3/8)\phi^4 + u\phi^6 \). A close examination of the renormalized effective potential shows that it has a minimum at nonzero \( \phi \) even for small positive values of \( r \) as long as the renormalized coupling \( \lambda \) is negative. As shown in [3], this nonzero charge condensate induces a
Meissner term in the effective action of the probing electromagnetic gauge fields which signals a superconducting state.

In summary, I evaluated the quantum corrections to the effective potential of a theory with dual electric and magnetic order parameters minimally coupled to topologically massive gauge fields. Such a theory has been shown to effectively describe the low-energy physics of a two-dimensional Josephson junction array [3]. The corrections to the effective potential due to gauge fluctuations are evaluated to one-loop order. It is shown that these fluctuations disfavor SSB, thus enhancing the stability of the insulating phase versus the superconducting one. The scenario drastically changes when massless fermions are coupled to the disorder field $\phi$. In this case, the effective potential gets modified by a contribution which leads to SSB and, as a result, the superconducting phase is now favored. This finding is in agreement with that of Ref. [5] where it is found that coupling of a JJA system to a graphene substrate with gapless excitations reduces quantum phase fluctuations and triggers a transition to the superconducting phase.