

Transmission and Reflection of Collective Modes in Spin-1 Bose-Einstein Condensation

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Abstract We study tunneling properties of collective excitations in spin-1 Bose-Einstein condensate. In the absence of the magnetic field, all kinds of excitations but the quadrupolar spin mode in the ferromagnetic state experience the perfect transmission in the long wavelength limit. We also discuss results in the presence of the magnetic field. We present an unified viewpoint of the perfect transmission of excitations in the Bose-Einstein condensate.

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1 Introduction

Bose-Einstein condensation (BEC) with internal degrees of freedom is known to have rich order parameter manifolds^{1,2}. Thanks to internal degrees of freedom, these systems also have several kinds of low-lying excitations^{1,2}. With successful experiments on BEC in optically trapped dilute gases, BEC with internal degrees of freedom has been extensively studied^{3,4,5}.

In the spin-1 BEC system, there are 2-types of the ground state, called ferromagnetic state and polar state, depending on an interaction parameter^{1,2}. For excitations, there are 3-types in each ground state, such as the Bogoliubov mode and the spin wave mode.

The purpose of the present study is to reveal transmission properties of those excitations in a spin-1 BEC against a potential barrier. This study proceeds from the tunneling problem of Bogoliubov excitations in a scalar BEC^{6,7}. A striking property in a scalar BEC is the perfect transmission of the Bogoliubov excitation in the low-energy limit. This problem has been extensively and intensively studied within the scalar BEC^{8,9,10,11,12}. A Natural question concerning spin-1 BEC is what kinds of excitations transmit perfectly, transmit partially or reflect perfectly. With these motivations, we shall study the tunneling problem of excitations in the spin-1 BEC.

Significance of the present study are the followings:

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(i) *Tunneling properties of excitations in spin-1 BEC is clarified.* Many studies of tunneling properties of a scalar BEC have been done, and revealed basic and unknown properties of BEC^{6,7,8,9,10,11,12}. On the other hand, tunneling properties of excitations in spin-1 BEC have been untouched problem. Many interesting properties would lie in this issue. The present paper is a first report on tunneling problems of excitations in spin-1 BEC.

(ii) *Unified view of tunneling properties in BEC in the low-energy limit is presented.* In the spin-1 BEC, 3-types of excitations exist^{1,2}. Those excitations are different properties in wave functions and in dispersion relations. However, there is a common background in the perfect transmission. A wavefunction of condensate plays an important role in these tunneling problem in the long wavelength limit, as in the problem in a scalar BEC⁹.

2 Formulation

We start with the following Gross-Pitaevskii (GP) equations for the spin-1 BEC system^{1,2}, describing the stationary solution of order parameters Φ_i for spin index $i = \pm 1, 0$. These equations in a simplified form¹³ are given by

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V_{\text{ext}} - \mu + c_0 n \mp g\mu_B B^z\right)\Phi_{\pm 1} + c_1\left(\frac{1}{\sqrt{2}}F_{\mp}\Phi_0 \pm F_z\Phi_{\pm 1}\right) = 0, \quad (1)$$

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V_{\text{ext}} - \mu + c_0 n\right)\Phi_0 + \frac{c_1}{\sqrt{2}}(F_+\Phi_{-1} + F_z\Phi_{-1}) = 0, \quad (2)$$

where $n = \sum_i |\Phi_i|^2$, $F_z = |\Phi_{+1}|^2 - |\Phi_{-1}|^2$, and $F_+ = F_-^* = \sqrt{2}(\Phi_{+1}^*\Phi_0 + \Phi_0^*\Phi_{-1})$. Interaction strengths are given by $c_0 = 4\pi\hbar^2(2a_2 + a_0)/(3m)$, and $c_1 = 4\pi\hbar^2(a_2 - a_0)/(3m)$, where m is a mass, and a_S is the s -wave scattering length with collision process with total spin S . g is the Landè's g -factor, μ_B is the Bohr magneton. In the case $c_1 < 0$, the ground state is so called the ferromagnetic state, while the ground state with $c_1 > 0$ is the polar state. In this article, we discuss the tunneling problem of excitations, and hence we shall consider small fluctuations from wavefunction of the condensates: $\Phi_i + \delta\Phi_i$.

In a homogeneous system, excitations of energy and wavefunctions have been clarified^{1,2}. We summarize physical properties of excitations in spin-1 BEC. In the ferromagnetic state, one mode is a Bogoliubov mode corresponding to the scalar BEC, where the fluctuation is $\delta\Phi_{+1}$ and the excitation spectrum is given by $E = \sqrt{\varepsilon[\varepsilon + 2(c_0 + c_1)n]}$. ε is given by $\varepsilon = \hbar^2 k^2/(2m)$. Another mode is a spin wave one, where the fluctuation is $\delta\Phi_0$ and the excitation energy is given by $E = \varepsilon + g\mu_B B^z$. The other mode is a quadrupolar spin mode², where the fluctuation is $\delta\Phi_{-1}$ with the energy $E = \varepsilon + 2|c_1|n + 2g\mu_B B^z$.

On the other hand, in the polar state in the absence of the magnetic field ($B^z = 0$), one mode is also a Bogoliubov mode, where the fluctuation is $\delta\Phi_0$ with $E = \sqrt{\varepsilon(\varepsilon + 2c_0n)}$. The others are spin wave modes doubly degenerated, where fluctuations are $\delta\Phi_{\pm 1}$ with $E = \sqrt{\varepsilon(\varepsilon + 2c_1n)}$.

Consider how much those excitations reflect and transmit when they run toward a potential wall. We follow calculations mentioned below. First, we solve the above Gross-Pitaevskii equations including the potential barrier, $V_{\text{ext}}(x) = V_0\theta(a - |x|)$, with the Heaviside's step function $\theta(x)$. Using obtained order parameters, we evaluate transmission and reflection coefficients by solving equations for small fluctuations, such as a Bogoliubov equation, with imposing the boundary conditions where incident, reflected and transmitted

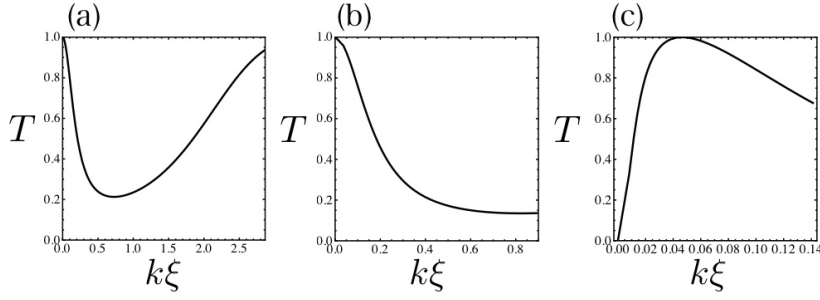


Fig. 1 Transmission coefficients of excitations in the ferromagnetic state as a function of the wavenumber. Each figure corresponds to (a) Bogoliubov excitation, (b) spin wave mode, and (c) quadrupolar spin mode at $B^z = 0$. ξ is given by $\xi \equiv \sqrt{2mc_0 n}/\hbar$. We set parameters $V_0 = 3c_0 n$ and $a = \xi/2$.

waves exist at asymptotic regimes. The details of formulation will be reported in another article. Preceding studies in a scalar BEC system are of some help for analysis on the present issue^{6,7,8,9,10,11,12}.

In the next section, we present results on transmission coefficients and phase shifts of excitations. Phase shift is defined by the argument of an amplitude transmission coefficient. First, we give results of the ferromagnetic state, and after that give ones of the polar state.

3 Results

3.1 Ferromagnetic state

In this subsection, we show results in the ferromagnetic state. We use parameters a_0 and a_2 as $a_0 : a_2 = 110 : 107$ as shown in Ref. 2. Figures 1 and 2 show transmission coefficients and phase shifts of excitations. In each figure, (a), (b) and (c) show the results of excitations described by the Bogoliubov mode, spin wave mode, and quadrupolar spin mode.

Note that excitations but the fluctuation of quadrupolar spin experience the perfect transmission in the long wavelength limit. Phase shifts also go to zero in the above two excitations. These behaviors are qualitatively the same as in the scalar BEC, regardless of dispersion relation. Unique behavior is seen in excitations in the fluctuations of the quadrupolar spin. Only this excitation experiences the perfect reflection, and the phase shift does not go to zero in the long wavelength limit. We find that the argument of an amplitude reflection coefficient is π in the long wavelength limit, and hence the quadrupolar spin mode experiences the fixed-end reflection. We have checked that results in the presence of the magnetic field ($B^z \neq 0$) are the same as ones at $B^z = 0$, shown above.

3.2 Polar state

In this subsection, we present results in the polar state at $B^z = 0$. We use parameters a_0 and a_2 as $a_0 : a_2 = 46 : 52$ as shown in Ref. 2. Figures 3 and 4 show transmission coefficients and phase shifts of excitations. In each figure, (a) and (b) show the results of the Bogoliubov

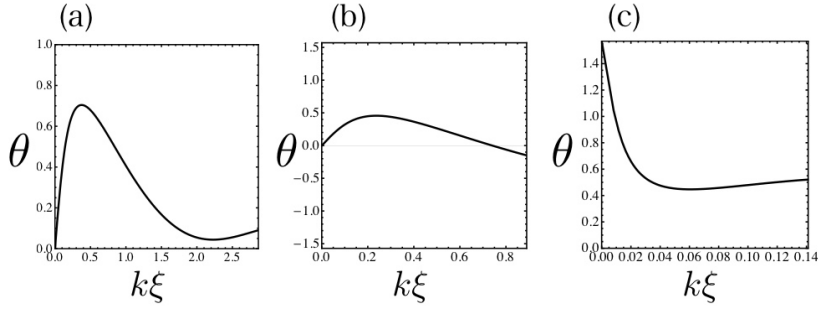


Fig. 2 Phase shifts of excitations in the ferromagnetic state as a function of the wavenumber. Each figure corresponds to (a) Bogoliubov excitation, (b) spin wave mode, and (c) quadrupolar spin mode at $B^z = 0$. We set parameters $V_0 = 3c_0n$ and $a = \xi/2$.

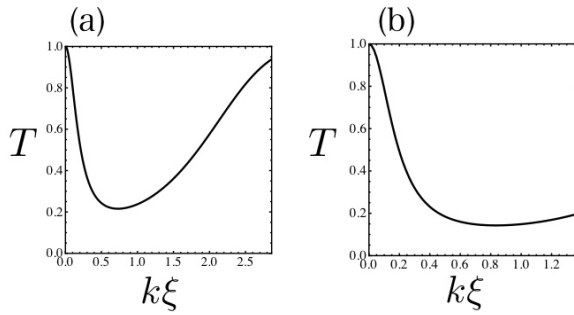


Fig. 3 Transmission coefficients of excitations in the polar state as a function of the wavenumber. Each figure corresponds to (a) Bogoliubov excitation, and (b) spin wave modes doubly degenerated at $B^z = 0$. We also use parameters $V_0 = 3c_0n$ and $a = \xi/2$.

excitation, and spin wave modes doubly degenerated, respectively. We note that all excitations experience the perfect transmission in the long wavelength limit. Phase shifts also go to zero in the long wavelength limit.

We note that, in polar state at $B^z \neq 0$, there is a special case $c_0 = c_1$. This special condition is known to be the integrable condition for spin-1 GP equation found by Ieda¹⁴. In this condition, excitations of ± 1 components from the ground state are completely decoupled, and hence the properties of excitations are the same as in the scalar BEC. We confirmed that 3-types of excitations in the case $c_0 = c_1$ experience the perfect transmission in the long wavelength limit.

4 Discussion

When one sees the above results, one finds almost all excitations can experience the perfect transmission in the long wavelength limit. Only the quadrupolar spin mode cannot transmit perfectly in the long wavelength limit. One of the issues is what kind of excitations can experience the perfect transmission in the long wavelength limit.

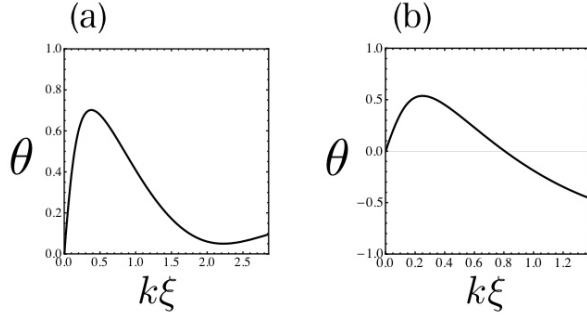


Fig. 4 Phase shifts of excitations in the polar state as a function of the wavenumber. Each figure corresponds to (a) Bogoliubov excitation, and (b) spin wave modes doubly degenerated at $B^z = 0$. We set parameters $V_0 = 3c_0n$ and $a = \xi/2$.

One of clue is an energy gap of excitation, because the quadrupolar spin mode only has the energy gap in the absence of the magnetic field. However, at $B^z \neq 0$, the spin wave mode in the ferromagnetic state still experiences the perfect transmission as shown in Fig. 5 (a), even though this spin wave mode also has the energy gap due to the Zeeman shift.

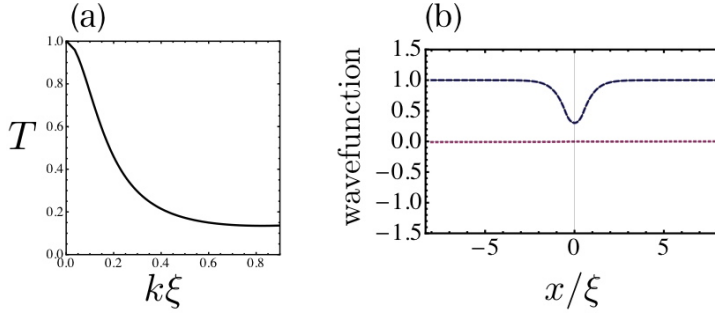


Fig. 5 (a) Transmission coefficient of the spin wave mode in the ferromagnetic state at $B^z \neq 0$. We added the magnetic field $g\mu_B B^z = 2c_0n$ to the system shown in Fig. 1. Although this excitation spectrum has the energy gap due to the Zeeman shift, this mode transmits perfectly in the long wavelength limit. (b) Wavefunction as a function of the position x in the ferromagnetic state at $B^z \neq 0$. The solid line represents condensate wavefunction Φ_{+1} . The dashed line represents a real part of the wavefunction of the spin wave mode in the long wavelength limit with energy $E = 10^{-7}c_0n + g\mu_B B^z$, where we set $g\mu_B B^z = 2c_0n$. The dotted line represents an imaginary part of the wavefunction of the spin wave mode in the long wavelength limit. Note that the wavefunction of the spin wave mode in the long wavelength limit coincides with the wavefunction of the condensate.

Another clue is the correspondence between the wavefunction of excitation and that of the condensate. On the Bogoliubov mode, the perfect transmission in the context of the anomalous tunneling is studied, and one of the authors found that the perfect transmission occurs when wavefunction of excitations corresponds to that of the condensate wavefunction in the low energy limit⁹. In the spinor BEC, this property also holds. For example, in the spin wave mode in the ferromagnetic state, the wavefunction in the long wavelength limit also corresponds to the condensate wavefunction despite the existence of the energy gap

due to the magnetic field, and also despite excitations with a parabolic spectrum, as shown in Fig 5 (b). We confirmed that all excitations showing the perfect transmission in the long wavelength limit studied in the present paper satisfy this correspondence.

From those results, we would make a conjecture, that *an excitation, whose wavefunction coincides with condensate wavefunction in the long wavelength limit, transmits perfectly against a potential barrier in the long wavelength limit, despite of the property of excitations.* We should remark that this is valid for the condensate wavefunction which is symmetric in asymptotic regimes. It is known that a partial transmission of the Bogoliubov excitation occurs, if we consider a case where the condensate wavefunction is asymmetric in asymptotic regimes¹⁰. It is an interesting future problem to reveal how excitations transmit in such an asymmetric system in spin-1 BEC.

5 Conclusion

We reported transmission properties of the excitations in the spin-1 BEC system against a potential barrier in some conditions. We showed transmission coefficients and phase shifts as a function of the wavelength. We found that all excitations but the quadrupolar spin mode transmit perfectly in the long wavelength limit. A correspondence between wavefunction of an excitation and that of condensate plays an important role in the perfect transmission. We expect that this problem blossoms to basic and extended issues, such as a tunneling problem of excitations through domain walls and topological defects.

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